RCK12 Middle and High School Mathematics Instructional Manual

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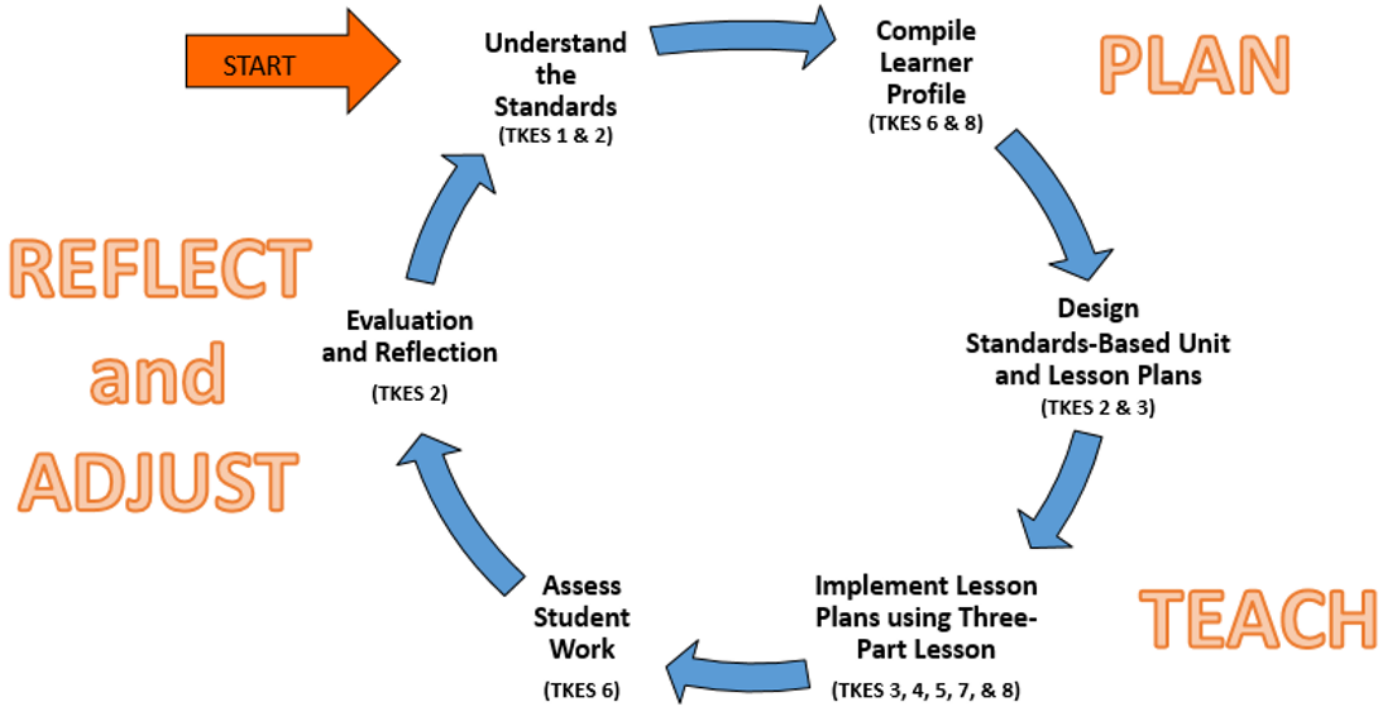
RCK12 Instructional Framework



PLAN

Understand the Standards (TKES Standard 1 & 2)

Review Learning Targets and Success Criteria for each Unit
Identify Key Vocabulary

Compile Learner/Class Profiles (TKES Standards 6 and 8)

Set Learning Goals for each Student
Adjust/Differentiate Instruction based on Quantile Data from *i-Ready*

Design Standards-Based Units and Lessons (TKES Standards 2 and 3)

Review District Developed Standards-Based Units
Review and/or Develop Pre and Post Assessments for the Unit based on the Learning Targets

TEACH

Implement Unit (TKES Standards 3, 4, 5, 7, 8)

Teach Three-Part Lesson that includes the 5Es and Formative Assessment
Provide interventions for Struggling Students
Enrich Students Who Have Met Standards

Assess Student Work (TKES Standard 6)

Analyze Student Work to Identify Strengths and Gaps
Provide Feedback

REFLECT and ADJUST

Evaluation and Reflection (TKES Standard 2)

Revisit Student Goals and Make Adjustments According to Student Assessment Data
Identify Interventions for Struggling Students
Identify Students Who Have Met Standards and Need Enrichment

Tiers of Instruction Overview

The intervention tiers are on a continuum that is fluid allowing students to move up or down the tiers throughout their educational careers. The student's level of need dictates the tier of support. The actual length of time that an intervention is implemented depends on the student's response to the intervention and the minimum requirements specified by the program.

Tier 1 *core* instruction is the instruction that all students in a classroom receive. It entails universal screening of all students, regardless of proficiency, using valid measures to identify students at risk for future academic failure—so that they can receive early intervention. Tier 1 instruction is "high quality."

Tier 2 *targeted group* interventions—schools provide additional assistance to students who demonstrate difficulties on screening measures or who demonstrate weak progress. Tier 2 students receive supplemental small group instruction aimed at building targeted proficiencies. Student progress is monitored throughout the intervention (no more than seven students per group).

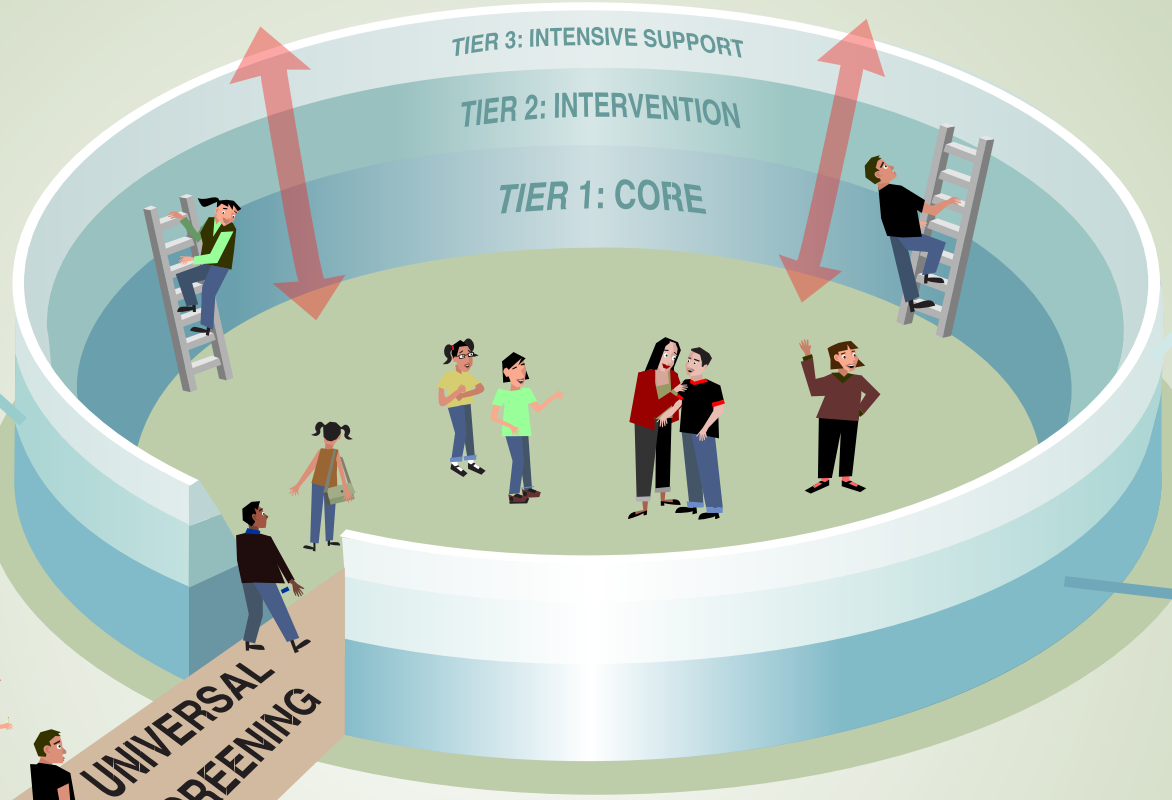
Tier 3 *intensive* interventions are provided to students who are not benefiting from tier 2 and require more intensive assistance. Tier 3 usually entails one-on-one instruction along with an appropriate mix of instructional interventions. Ongoing analysis of student performance data is critical in this tier (no more than 3 students per group).

Tier 4 is the *most intensive* tier of instruction for students identified as eligible for Special Education. Students at this level receive specially designed instruction as determined by the Special Education Department.

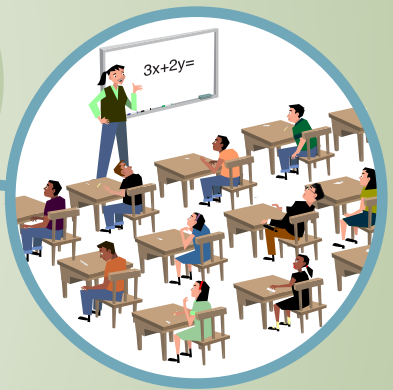
RCK12 Response to Intervention Framework in Mathematics

Response to Intervention (RtI) is a framework for supporting students who are potentially at risk and assisting them before they fall behind. RtI is grounded in high quality core classroom instruction for all students which is then supplemented as necessary by progressively more intensive interventions for students who may struggle with mathematics. Key components of RtI are periodic universal screening to determine which students may need additional instruction and ongoing progress monitoring to ascertain the effectiveness of additional instruction.

Tier 2: Interventions
Small group, more intense Instruction focused on specific skills



Tier 3: Intensive Support
Most intense, often one-to-one



Tier 1: Core Instruction
All students

All Students

Screening and Monitoring

- Screening three times a year (iReady)
- Ongoing progress monitoring (iReady)
- Reliable, efficient, valid measures

Tiers 2 & 3

Foundations of Arithmetic

- In-depth topic coverage:
  - Whole numbers (K-5)
  - Rational numbers (4-9)
- Underlying structures of word problems
- Daily fact fluency practice

Tiers 2 & 3

Intentional Teaching

- Explanations, think alouds, modeling
- Scaffolded problem-solving
- Visual representations
- Immediate corrective feedback
- Praise for effort & engagement

DOING WHAT WORKS | http://dww.ed.gov
U.S. Department of Education (modified for RCSS)

RCK12 Universal Screening and Progress Monitoring Recommendations

Screen all students to identify those at risk for potential math difficulties and provide interventions to students identified as at risk.

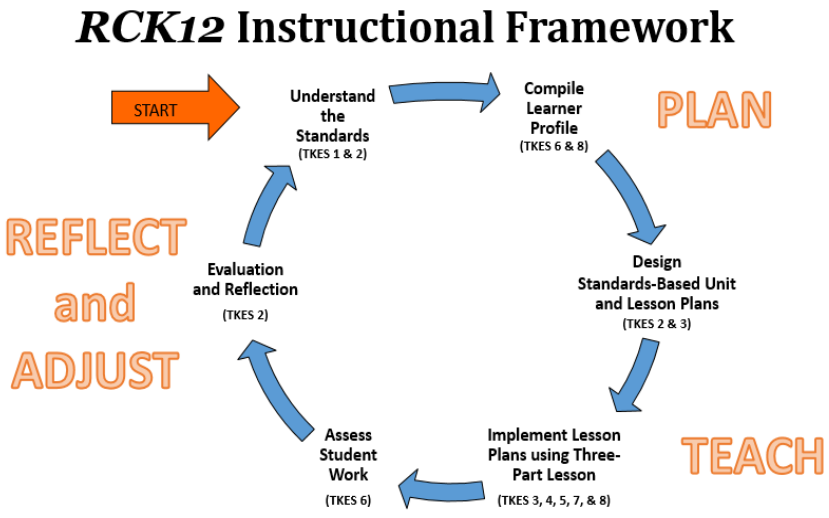
The recommendations for Progress Monitoring in math are based on the information teachers get from the diagnostic assessments, and the progress monitoring recommendations are outlined below.

- Tier 3 students (Red): every two weeks
- Tier 2 students (Yellow): once a month
- Tier 1 students (Green): at least once each 9 weeks (teacher discretion; as needed)

The measures for progress monitoring – and subsequent screenings – are equivalent in difficulty to the assessments given at the beginning of year. Progress is gauged by administering the same measures over time and tracking if students are able to perform on grade level in reading and math.

How is Progress Monitoring Connected to Instruction?
The Richmond County Instructional Framework supports setting goals for students utilizing instructional data and making adjustments based upon progress monitoring.

Progress monitoring is directly tied to instruction, so teachers should focus first on creating and implementing an instructional plan for each student. Progress monitoring results are critical tool in making these adjustments as they will tell if student's growth is indeed occurring and if it is at the appropriate rate for students to be on track by the next benchmark assessment period.



When identifying the area in which students need instruction, it is critical that teachers review all math and Quantile measures. The student's score in each measure provides focus on the most critical areas for student's growth. Students who score more than one grade level below should receive intensive intervention on a continual basis.

RCK12 Pyramid of Interventions for Mathematics

Assessment Resources:

Universal Screeners

Grade	Screener	Progress Monitoring
K – 9th	i-Ready	i-Ready

Other Assessments

District Performance/Culminating Task (K – 12)

RCK12 Curriculum Unit Pre and Post Assessment (K – 12)

RCK12 Curriculum Unit Informal Check Assessments (K – 12)

Strategies for Tier I Mathematics Instruction:

- Concrete-Representational-Abstract Model
- Graphic Organizers
- Review of prior math concepts (includes spiraling lessons and homework)
- Strategy instruction (includes teacher modeling; think-aloud models and cues; simplifying and reducing strategies)
- Hands-on devices and visual aids (includes manipulatives; graphing calculators; graph paper; flash cards; drawings; number lines)
- Cooperative learning (includes small-group instruction; teaming; peer-assisted tutoring)
- Goal setting/structure
- Scaffolding (includes guided teacher practice and questioning)
- Simulations (includes role-playing or acting out mathematical concepts)
- Computer-assisted instruction

Please note that these are examples of interventions at each tier and not all inclusive. Some interventions may be applicable to more than one tier depending on the intensity and /or frequency of the intervention.

Tier 4 - Special Education

Tier 3

SST DRIVEN INSTRUCTION
Small Group (no more than 3 students)
(Individually assigned/needs based)

K – 5th envision Math: Intervention System
Numeracy Development Project Books*
6th – 9th i-Ready Intervention Print Lessons
Numeracy Development Project Books*
*located in RCK12 Mathematics Curriculum (Rubicon Atlas)

Progress Monitoring every two weeks

Tier 2

DATA DRIVEN TARGETED INSTRUCTION
Small Group (no more than 7 students)
(Individually assigned/needs based)

K – 5th i-Ready Intervention Print Lessons and online instruction, Targeted First in Math RTI, **GaDOE Intervention Table***
6th – 8th i-Ready Intervention Print Lessons and online instruction, Go Math RTI Intervention System, **GaDOE Intervention Table***
9th – 11th i-Ready Intervention Print Lessons and online (9th grade ONLY), HRW (AGA) RTI Intervention System

*located in RCK12 Mathematics Curriculum (Rubicon Atlas)

Progress Monitoring once monthly

Tier 1

STANDARDS-BASED MATHEMATICS INSTRUCTION

K – 5th RCK12 Curriculum (rcboe.rubiconatlas.org), enVision Math Instructional Resource/Textbook
Think Math with lessons focused on Concrete-Representational-Abstract Model
First in Math Software, Quantile Framework for Mathematics (https://www.quantiles.com)

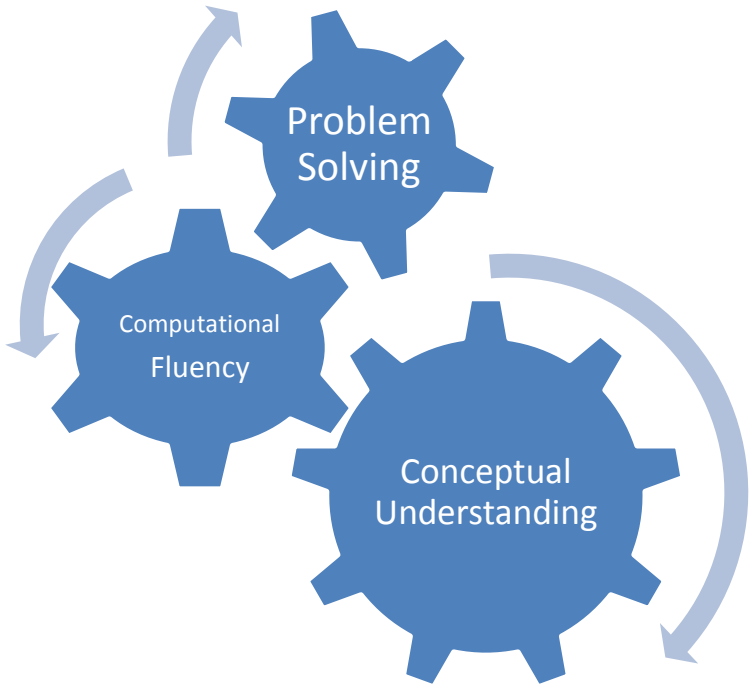
6th – 8th RCK12 Curriculum (rcboe.rubiconatlas.org), GO Math Instructional Resource/Textbook
Georgia Virtual School Modules, Edgenuity lessons, Quantile Framework for Mathematics (https://www.quantiles.com)

9th – 12th RCK12 Curriculum (rcboe.rubiconatlas.org), HRW (AGA) Collections AND Pearson Collections Instructional Resource/Textbook, Georgia Virtual School Modules, Edgenuity lessons, Quantile Framework for Mathematics – 9th grade ONLY (https://www.quantiles.com)

Tier 1
Instructional Expectations

RCK12 Balanced Numeracy

The Richmond County School System's Mathematics Department provides an aligned, standards-based, **balanced**, and engaging comprehensive curriculum including Curriculum Maps, Unit Snapshots, Unit Pacing Guides, Pre/Post Assessments, Informal Checks, Performance Tasks, Culminating Tasks, and five days of scripted lesson plans for each unit. These documents specify the standards to be taught for each grade level that will not only effectively prepare students to be College and Career Ready but also instill a passion for mathematics. The **RCSS Curriculum** and **Balanced Numeracy** approach addresses the following key components:



1. **Conceptual Understanding:** The conceptual understanding part of the math lesson is designed to explore, develop, and teach mathematical concepts using the Concrete-Representational-Abstract Model (i.e. math tools).

2. **Problem Solving:** Problem Solving develops mathematical reasoning and problem solving abilities. Problem solving provides students the opportunity to apply the mathematics they are learning in the Conceptual Understanding component of instruction to a problem-solving situation.

3. **Computational Fluency**
   - **Math Review:** Math review emphasizes the development of number sense as students practice procedural mathematics and computational skills every day. Learning objectives are connected to what students have previously learned. Student work connects what they are learning to prior learning.
   - **Mental Math:** Mental math helps students become skillful in computing math problems mentally.

RCK12 Balanced Numeracy "Looks-Fors"

Conceptual Understanding (Know WHY)	Computational Fluency (Know HOW)	Problem Solving (Know WHEN)
Concrete-Representational-Abstract • **Concrete** math tools (i.e. base-ten blocks, cubes, counters) • Pictorial **representations** (i.e. circles to represent coins, pictures of objects, tally marks, number lines) • Numbers and variables to explain how symbols can be used as an efficient way to represent **numeric and algebraic** situations • Show different representations of the same mathematical situation	***Math Review and Mental Math*** • 2-5 problems daily on the same repeat concept • Emphasis on Number Sense (reasonable answers and estimation) • Error Analysis • Student Reflection (student talk) • Problem strings of numbers and operations (i.e. Number Talks and Problem Strings) • Strategies to solve problems	***Mathematical Reasoning*** • Understand and explore the problem • Strategies to solve problems • Look back and reflect on the solution • Connect new learning to prior knowledge to make sense of the problem • Apply conceptual learning to familiar and unfamiliar situations

RCK12 Balanced Numeracy Classroom Expectations

Numeracy Block	Opening	Work Session	Closing
The Numeracy Block starts with Number Talks, Calendar Math, and/or Fluency Activities to build computational fluency using a variety of strategies.	Explicit whole class guided (teacher modeling) and shared instruction aligned to the standards that includes a balance of: • Computational Fluency (i.e. purposeful practice) • Conceptual Understanding (i.e. using tools) • Problem Solving (i.e. problems that students can solve in a variety of ways)	Data-driven independent, partner, and small group scaffolded instruction that provides students with opportunities to engage in: • Problem Solving • Purposeful Fluency Practice • Conceptual Understanding using concrete and pictorial representations • Explicit Teacher Instruction for addressing individual needs, etc. Teacher and peers conference and provide timely feedback.	Purposeful Reflection • Encourage students to reflect on what they have learned, how they learned, and what assisted them in their learning.

Standards of Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

#1 Make sense of problems and persevere in solving them **What it means:** Understand the problem, find a way to attack it, and work until it is done. Basically, you will find practice standard #1 in every math problem, every day. The hardest part is pushing students to solve tough problems by applying what they already know and to monitor themselves when problem-solving. **Own it:** Give students tough tasks and let them work through them. Allow wait time for yourself and your students. Work for progress and "aha" moments. The math becomes about the process and not about the one right answer. Lead with questions, but don't pick up a pencil. Have students make headway in the task themselves.	**#2 Reason abstractly and quantitatively** **What it means:** Get ready for the words *contextualize* and *decontextualize.* If students have a problem, they should be able to break it apart and show it symbolically, with pictures, or in any way other than the standard algorithm. Conversely, if students are working a problem, they should be able to apply the "math work" to the situation. **Own It:** Have students draw representations of problems. Break out the manipulatives. Let students figure out what to do with data themselves instead of boxing them into one type of organization. Ask questions that lead students to understanding. Have students draw their thinking, with and without traditional number sentences.
#3 Construct viable arguments and critique the reasoning of others **What it means:** Be able to talk about math, using mathematical language, to support or oppose the work of others. **Own it:** Post mathematical vocabulary and make your students use it — not just in math class, either! Use "talk moves" to encourage discourse. Work on your classroom environment from day one so that it is a safe place to discuss ideas.	**#4 Model with mathematics** **What it means:** Use math to solve real-world problems, organize data, and understand the world around you. **Own it:** Math limited to math class is worthless. Have students use math in science, art, music, and even reading. Use real graphics, articles, and data from the newspaper or other sources to make math relevant and real. Have students create real-world problems using their mathematical knowledge.

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#5 Use appropriate tools strategically **What it means:** Students can select the appropriate math tool to use and use it correctly to solve problems. In the real world, no one tells you that it is time to use the meter stick instead of the protractor. **Own it:** Don't tell students what tool to use. Try to leave the decision open ended and then discuss what worked best and why. For example, I wanted my students to find their height. They had measuring tapes, rulers, and meter sticks among their math tools. Once everyone found their height, we discussed which tools worked best and why. Leave math tools accessible and resist the urge to tell students what must be used for the task. Let them decide; they might surprise you!	**#6 Attend to precision** **What it means:** Students speak and solve mathematics with exactness and meticulousness. **Own it:** Push students to use precise and exact language in math. Measurements should be exact, numbers should be precise, and explanations must be detailed. One change I've made is not allowing the phrase, "I don't get it." Students have to explain exactly what they do and do not understand and where their understanding falls apart.
#7 Look for and make use of structure **What it means:** Find patterns and repeated reasoning that can help solve more complex problems. For young students this might be recognizing fact families, inverses, or the distributive property. As students get older, they can break apart problems and numbers into familiar relationships. **Own It:** Help students identify multiple strategies and then select the best one. Repeatedly break apart numbers and problems into different parts. Use what you know is true to solve a new problem. Prove solutions without relying on the algorithm. For example, my students are changing mixed numbers into improper fractions. They have to prove to me that they have the right answer without using the "steps."	**#8 Look for and express regularity in repeated reasoning** **What it means:** Keep an eye on the big picture while working out the details of the problem. You don't want kids that can solve the one problem you've given them; you want students who can generalize their thinking. **Own it:** Show students how the problem works. As soon as they "get it," start making them generalize to a variety of problems. Don't work fifty of the same problem; take your mathematical reasoning and apply it to other situations.

Reference: http://www.scholastic.com/teachers/top-teaching/2013/03/guide-8-mathematical-practice-standards

Tier 2 - 3

Instructional Expectations

Seven Recommendations for Mathematics Interventions

This manual offers seven recommendations for supporting students struggling in mathematics. The recommendations are intended to be implemented within an RtI framework (four tiers for Georgia). For RtI tiers 2 and 3, recommendations 1 through 7 focus on the most effective content and pedagogical practices that can be included in mathematics interventions.

1. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of rational numbers in grades 6 through 9 (refer to the Pyramid of Interventions for Tier 2 and Tier 3).

2. Instruction during the intervention should be explicit and systematic. This includes providing models of problem solving, verbalization of thought processes (i.e. think alouds), guided practice (i.e. gradual release model), corrective feedback, and frequent cumulative review.

3. Interventions should include instruction on solving word problems that is based on common underlying structures.

4. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas, and interventionists should be proficient in the use of visual representations of mathematical ideas (i.e. Concrete-Representational-Abstract Model).

5. Interventions at all grade levels should devote time in each session to building fluent retrieval of basic arithmetic facts (i.e. computational strategies using mental math).

6. Monitor the progress of students receiving supplemental instruction and other students who are at risk.

7. Include motivational strategies in Tier 2 and Tier 3 interventions (refer to Growth Mindset Resources in Rubicon).

Reference: IES Practice Guide: Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools, **April 2009**

Small Group Interventions: Intentional Teaching "Look-Fors"

Math coaches and administrators can use this tool to observe and provide feedback to interventionists or classroom teachers regarding their use of intentional teaching pedagogy in small-group interventions. Math teachers can also use this form as a self-reflection on current practice and ways to improve their use of selected instructional practices.

Intentional Teaching Practices
1. The teacher provides step-by-step demonstrations and modeling of math concepts.
2. The teacher explains the reasoning behind each step, using "think alouds."
3. The teacher provides guided practice, including scaffolding and solving problems along with students.
4. The teacher frequently checks for student understanding.
5. The teacher asks students to explain their reasoning and approaches.
6. The teacher reteaches when students don't understand.
7. Reteaching includes demonstrations in alternate formats so students have multiple ways to understand and practice the concepts.
8. The teacher provides immediate corrective feedback to students, specifically identifying correct work and errors.
9. When students are ready, the teacher allows independent practice so students' understanding can be checked.
10. Students work on a varied mix of challenging and easier problems, including worked examples for students to review.
11. The teacher provides frequent review during the lesson.
12. The teacher summarizes key concepts as closure to the lesson.
13. The teacher employs visual representations and manipulatives to make math concepts explicit.
14. The teacher scaffolds students from concrete materials to representations to abstract concepts, spending adequate time to ensure student understanding at each stage.
15. The teacher encourages students to persist with challenging problems.
16. The teacher praises students' accomplishments and recognizes their efforts.

Teacher Self-Assessment Inventory: Working with Struggling Students

This checklist is designed for K-12 teachers of mathematics to use to reflect on how they are currently supporting students who are **struggling with learning mathematics** and identify opportunities for providing additional support.

Classroom Instruction		✓
17. In each lesson I demonstrate to students how to solve problems.	Step-by-step modeling	
	Think alouds (sharing how I address a problem)	
	Demonstrating more than one way to address a problem	
	Explicit directions and explanations	
18. I demonstrate multiple examples of similar problems.	Examples that are similar	
	Examples with slight variations	
	Alternating challenging problems with easier problems	
19. I use multiple representations in demonstrations.	Concrete materials	
	Representational materials	
	Story contexts	
	Visual diagrams	
	Virtual demonstrations (via computer)	
	Number line	
Instruction: What might you add to your instruction or do more frequently to provide clearer instruction to students who are struggling?		

Student Practice		✓
1. I encourage students to think aloud while solving problems.	Explain work to teacher	
	Document steps taken in problem solving	
	Using drawing or models to represent thinking	
	Write about their work	
2. I provide many opportunities for practice.	Guided practice (students work a problem and approach is checked)	
	Worked examples are included for independent practice	
	In-class independent practice	
3. I encourage students to explain their solutions to others.	Working with a partner	
	Working in cooperative groups	
	Demonstrating worked solutions to class	
4. I provide opportunities for students to develop fluency with arithmetic facts so that they can use working memory for more challenging aspects of problem solving	Purposeful practice	
	Encouragement of fact learning	
	Use of technology-based tutorials for purposeful practice	
Student Practice: How might you augment the practice opportunities you provide to struggling students?		

Formative Assessment		✓
1. I use formal formative assessments weekly to assess whether students are grasping concepts and skills.	Quizzes	
	Unit tests or portions of unit tests	
	Software-based diagnostics/assessments	
2. I use informal formative assessments to track students' understanding in each lesson.	Checking work by having students do work at board or on white boards	
	Warm-up drills	
	Exit or summary check-in's	
	Observation and informal questioning (frequently)	
3. I encourage self-assessment of whether students' are understanding concepts and skills.	Encouraging checking through estimation	
	Use of journals to record progress	
	Self-grading	
	Encouraging question asking	
Formative Assessment: Are there ways you can use formative assessment more effectively?		

Feedback and Communications		✓
1. I provide feedback immediately to students about the accuracy of their approaches and responses, including identifying the steps in problem solving that are not correct.	Correct accuracy of responses	
	Review all completed work, including homework	
	Expect correct responses	
2. I acknowledge students' efforts and persistence.	Providing direct messages about effort expended "Grading" effort as well as accuracy	
	Report to parents on effort	
	Clearly stated expectations for students	
3. I provide students information about the expandability of ability through practice	Modeling of effort involved in solving challenging problems	
	Reminding students of when they have gained skill through effort	
	Information to parents about their messages on ability	
Feedback and Communications: How might you augment or modify the types of feedback you provide to students?		

Intervention CONTENT Expectations

Grade Level	Intervention Content Focus	Grade Level Fluency Expectation
Fifth	Focus on rational numbers should include understanding the meaning of fractions, decimals, ratios, and percents, using visual representations (including placing fractions and decimals on number lines, and solving problems with fractions, decimals, ratios, and percents).	Multi-digit whole number multiplication (up to 3 digit by 2-digit factor) and division (up to 4 digit dividends and 2 digit divisors)
Sixth		Multi-digit division & Multi-digit decimal operations
Seventh	Focus on rational numbers should include understanding the meaning of fractions, decimals, ratios, percents and **integers**, using visual representations (including placing fractions and decimals on number lines, and solving problems with fractions, decimals, ratios, and percents).	Add/subtract/multiply/divide integers Solve one- and two-step algebraic equations
Eighth		Solve one- and two-step algebraic equations Solve simple 2×2 systems by inspection
Algebra I		Solve multi-step algebraic equations Solve 2×2 systems by inspection
Geometry	Focus on transformations and proportional reasoning to develop a formal understanding of similarity and congruence (including identify criteria for similarity and congruence of triangles, develop conceptual understanding with geometric proofs).	Triangle congruence and similarity criteria Use of coordinate and construction tools
Algebra II	Focus on relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers (including perform operations with complex numbers and solve equations); structural similarities between the system of polynomials and the system of integers (including draw on analogies between polynomial arithmetic and base-ten computation).	Divide polynomials with remainder by inspection (in simple cases). See structure in expressions and use this structure to rewrite expressions. Translating between recursive definitions and closed forms

Fluency: Procedural fluency is defined as skill in carrying out procedures *flexibly, accurately, efficiently*, and *appropriately*. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. *Fluency is based on a deep understanding of quantity and number*.

Fluent students:

- flexibly use a combination of deep understanding and number sense.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf

Reference: Georgia Department of Education, Georgia Standard of Excellence Math Overview, **2016**